Midsegment Theorem (pp 287-289)

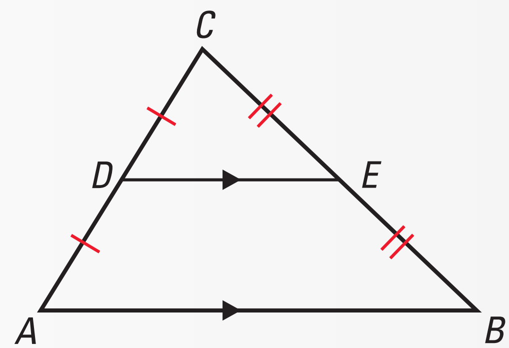
- I can define an identify the midsegments of a triangle.
- I can use the property of the midsegment of a triangle to solve problems.

THEOREM

THEOREM 5.9 ***Midsegment Theorem***

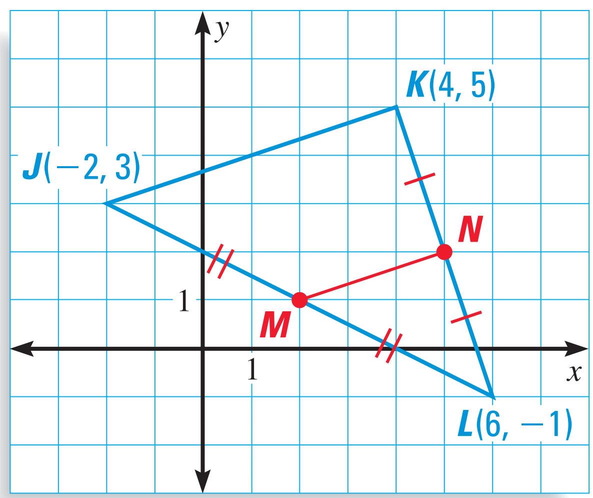
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

$\overline{DE} \parallel \overline{AB}$ and $DE = \frac{1}{2}AB$



EXAMPLE 1 *Using Midsegments*

Show that the midsegment $\overline{MN}$ is parallel to side $\overline{JK}$ and is half as long.



SOLUTION

Use the Midpoint Formula to find the coordinates of M and N.

$$M = \left(\frac{-2+6}{2}, \frac{3+(-1)}{2}\right) = (2, 1)$$

$$N = \left(\frac{4+6}{2}, \frac{5+(-1)}{2}\right) = (5, 2)$$

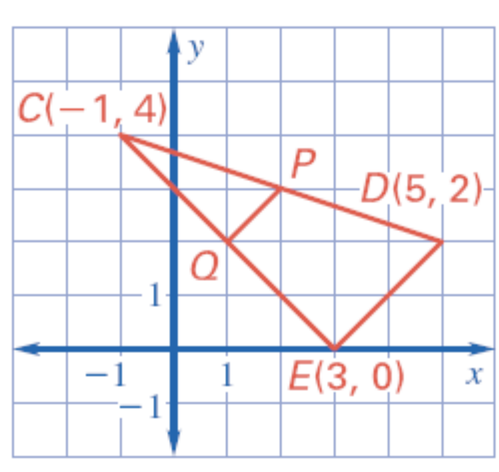
Next, find the slopes of $\overline{JK}$ and $\overline{MN}$.

Slope of $\overline{JK} = \frac{5-3}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$ Slope of $\overline{MN} = \frac{2-1}{5-2} = \frac{1}{3}$

▶ Because their slopes are equal, $\overline{JK}$ and $\overline{MN}$ are parallel. You can use the Distance Formula to show that $MN = \sqrt{10}$ and $JK = \sqrt{40} = 2\sqrt{10}$. So, $\overline{MN}$ is half as long as $\overline{JK}$.

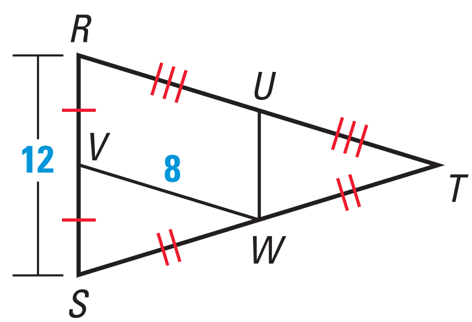
Midsegment Theorem (pp 287-289)

1. Show that midsegment $\overline{PQ} \parallel \overline{DE}$ and is half as long.



EXAMPLE 2 Using the Midsegment Theorem

$\overline{UW}$ and $\overline{VW}$ are midsegments of $\triangle RST$. Find UW and RT.

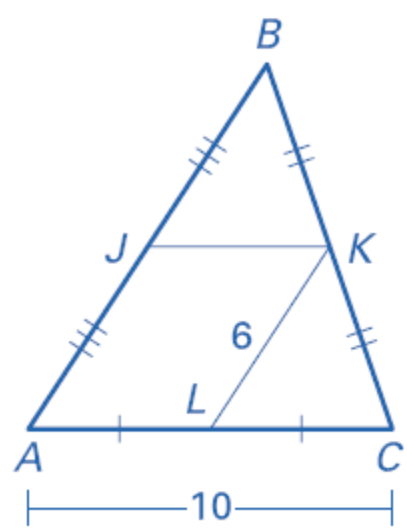


SOLUTION

$$UW = \frac{1}{2}(RS) = \frac{1}{2}(12) = 6$$

$$RT = 2(VW) = 2(8) = 16$$

2. $\overline{JK}$ & $\overline{KL}$ are midsegments of $\triangle ABC$. Find JK & AB.



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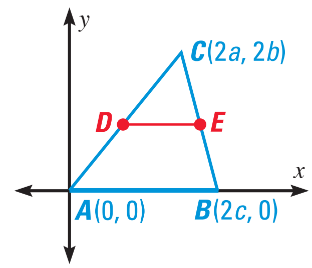
Midsegment Theorem (pp 287–289)

EXAMPLE 3 ***Proving Theorem 5.9***

Proof

Write a coordinate proof of the Midsegment Theorem.



SOLUTION

Place points A, B, and C in convenient locations in a coordinate plane, as shown. Use the Midpoint Formula to find the coordinates of the midpoints D and E.

$$D = \left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) = (a, b) \qquad E = \left(\frac{2a+2c}{2}, \frac{2b+0}{2}\right) = (a+c, b)$$

> **STUDENT HELP**
>
> **Study Tip**
> In Example 3, it is convenient to locate a vertex at (0, 0) and it also helps to make one side horizontal. To use the Midpoint Formula, it is helpful for the coordinates to be multiples of 2.

Find the slope of midsegment $\overline{DE}$. Points D and E have the same y-coordinates, so the slope of $\overline{DE}$ is zero.

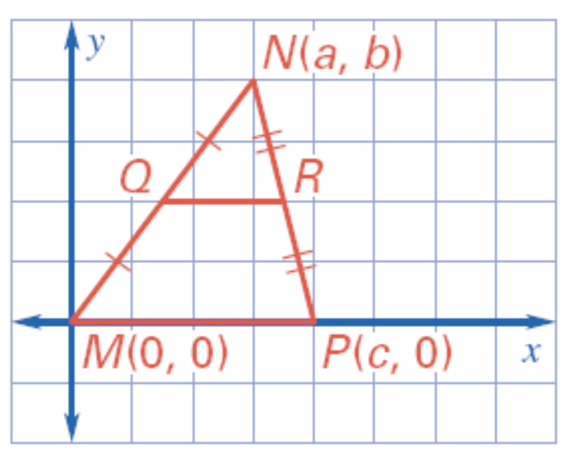
▶ $\overline{AB}$ also has a slope of zero, so the slopes are equal and $\overline{DE}$ and $\overline{AB}$ are parallel.

Calculate the lengths of $\overline{DE}$ and $\overline{AB}$. The segments are both horizontal, so their lengths are given by the absolute values of the differences of their x-coordinates.

$$AB = |2c - 0| = 2c \qquad DE = |a + c - a| = c$$

▶ The length of $\overline{DE}$ is half the length of $\overline{AB}$.

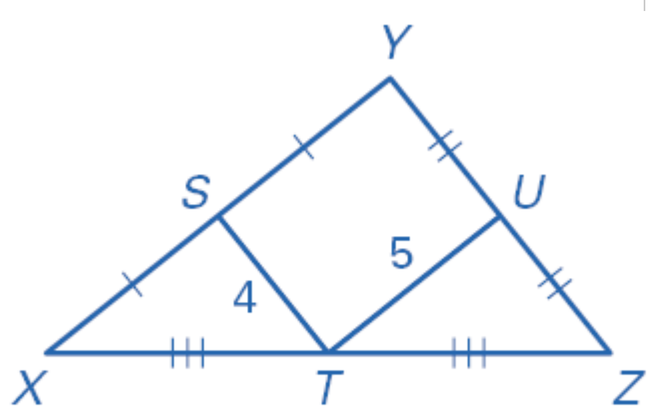
3. What are the coordinates of Q & R?



4. How do you know that $\overline{QR} \parallel \overline{MP}$?

5. What is the MP? What is QR?

6. In $\triangle XYZ$, which segment is parallel to $\overline{XY}$?



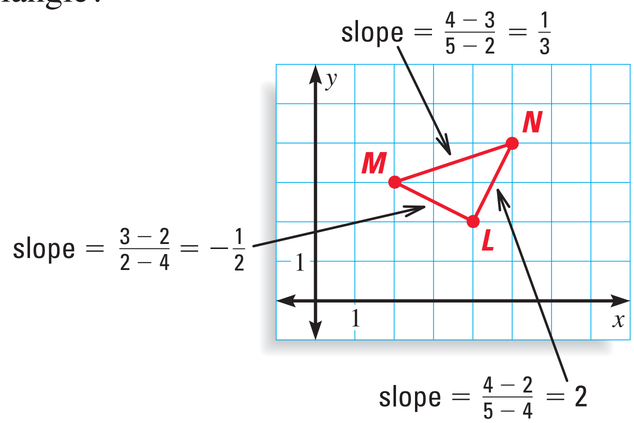
7. What is YZ?

Midsegment Theorem (pp 287-289)

Using Algebra

EXAMPLE 4 Using Midpoints to Draw a Triangle

The midpoints of the sides of a triangle are $L(4, 2)$, $M(2, 3)$, and $N(5, 4)$. What are the coordinates of the vertices of the triangle?



SOLUTION

Plot the midpoints in a coordinate plane.

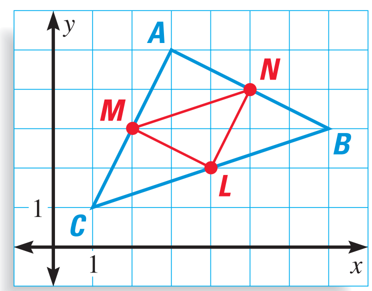
Connect these midpoints to form the midsegments $\overline{LN}$, $\overline{MN}$, and $\overline{ML}$.

Find the slopes of the midsegments. Use the slope formula as shown.

Each midsegment contains two of the unknown triangle's midpoints and is parallel to the side that contains the third midpoint. So, you know a point on each side of the triangle and the slope of each side.

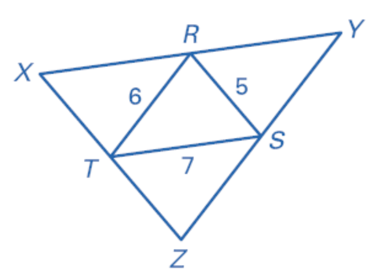
Draw the lines that contain the three sides.

▶ The lines intersect at $A(3, 5)$, $B(7, 3)$, and $C(1, 1)$, which are the vertices of the triangle.



8. The midpoints of the sides of a triangle are (1, 5), (3, 3), and (4, 6). What are the coordinates of the vertices of the triangle?

9. $\overline{RS}$, $\overline{ST}$, & $\overline{RT}$ are midsegments of $\triangle XYZ$. Find the perimeter of $\triangle XYZ$.



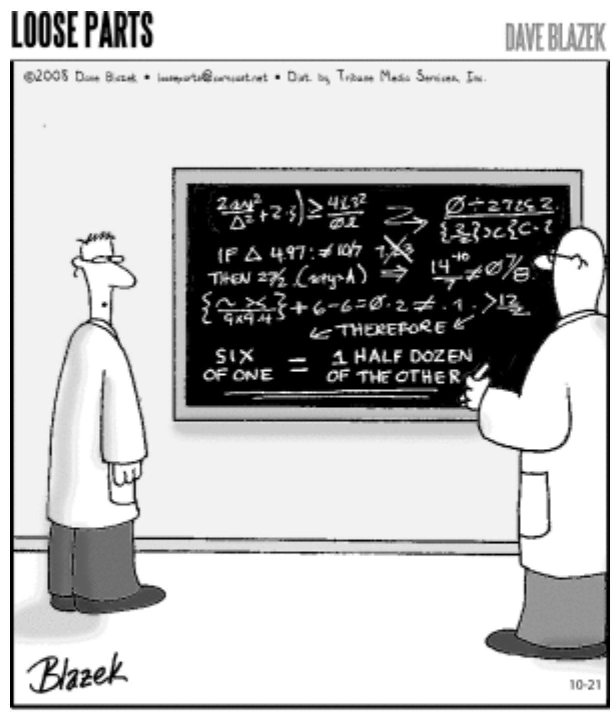
Midsegment Theorem (pp 287-289)

The midpoints of the sides of a triangle are (2, 5), (2, 2) & (6, 5).

10. What are the coordinates of the vertices of the triangle?

11. What is the perimeter of the triangle?

12. In $\triangle ABC,$ if M is the midpoint of $\overline{AB},$ N is the midpoint of $\overline{AC}$, & P is the midpoint of $\overline{BC},$ then $\overline{MN}, \overline{NP}, \& \overline{PN}$ are ______ of $\triangle ABC.$



"I must say, Giddings, it's a brilliant follow-up to your *Stitch In Time Saves Nine* proof."